35th BALKAN MATHEMATICAL OLYMPIAD
Belgrade, Serbia (May 9, 2018)

Language: English

Problem 1.

A quadrilateral $ABCD$ is inscribed in a circle k, where $AB > CD$ and AB is not parallel to CD. Point M is the intersection of the diagonals AC and BD and the perpendicular from M to AB intersects the segment AB at the point E. If EM bisects the angle CED, prove that AB is a diameter of the circle k.

Problem 2.

Let q be a positive rational number. Two ants are initially at the same point X in the plane. In the n-th minute ($n = 1, 2, \dots$) each of them chooses whether to walk due north, east, south or west and then walks the distance of q^n metres. After a whole number of minutes, they are at the same point in the plane (not necessarily X), but have not taken exactly the same route within that time. Determine all possible values of q.

Problem 3.

Alice and Bob play the following game: They start with two non-empty piles of coins. Taking turns, with Alice playing first, each player chooses a pile with an even number of coins and moves half of the coins of this pile to the other pile. The game ends if a player cannot move, in which case the other player wins.

Determine all pairs (a, b) of positive integers such that if initially the two piles have a and b coins respectively, then Bob has a winning strategy.

Problem 4.

Find all primes p and q such that $3p^{q-1} + 1$ divides $11^p + 17^p$.

Time allowed: 4 hours and 30 minutes.
Each problem is worth 10 points.