Serbia and Montenegro Team Selection Test 2006

Vršac, April 16, 2006

Time allowed 3 *hours.*
Each problem is worth 25 *points.*

1. The set $S = \{1, 2, 3, \ldots, 2006\}$ is partitioned into two disjoint subsets A and B such that:

   (i) $13 \in A$;

   (ii) if $a \in A$, $b \in B$, $a + b \in S$, then $a + b \in B$;

   (iii) if $a \in A$, $b \in B$, $ab \in S$, then $ab \in A$.

   Determine the number of elements of A.

2. A point P is taken in the interior of a right triangle ABC with $\angle C = 90^\circ$ such hat $AP = 4$, $BP = 2$, and $CP = 1$. Point Q symmetric to P with respect to AC lies on the circumcircle of triangle ABC. Find the angles of triangle ABC.

3. Determine all natural numbers n and $k > 1$ such that k divides each of the numbers

$$\binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}.$$