41-st International Mathematical Olympiad

Taejon, South Korea, July 13–25, 2000

First Day – July 18

1. Two circles G_1 and G_2 intersect at M and N. Let AB be the line tangent to these circles at A and B, respectively, such that M lies closer to AB than N. Let CD be the line parallel to AB and passing through M, with C on G_1 and D on G_2. Lines AC and BD meet at E; lines AN and CD meet at P; lines BN and CD meet at Q. Show that $EP = EQ$. *(Russia)*

2. Let a, b, c be positive real numbers with product 1. Prove that
$$\left(a - 1 + \frac{1}{b}\right)\left(b - 1 + \frac{1}{c}\right)\left(c - 1 + \frac{1}{a}\right) \leq 1.$$
(United States of America)

3. Let $n \geq 2$ be a positive integer and λ a positive real number. Initially there are n fleas on a horizontal line, not all at the same point. We define a move of choosing two fleas at some points A and B, with A to the left of B, and letting the flea from A jump over the flea from B to the point C such that $BC/AB = \lambda$.
Determine all values of λ such that for any point M on the line and for any initial position of the n fleas, there exists a sequence of moves that will take them all to the position right of M. *(Belarus)*

Second Day – July 19

4. A magician has one hundred cards numbered 1 to 100. He puts them into three boxes, a red one, a white one, and a blue one, so that each box contains at least one card. A member of the audience draws two cards from two different boxes and announces the sum of numbers on those cards. Given this information, the magician locates the box from which no card has been drawn. How many ways are there to put the cards in the three boxes so that the trick works? *(Hungary)*

5. Does there exist a positive integer n such that n has exactly 2000 prime divisors and $2^n + 1$ is divisible by n? *(Russia)*

6. $A_1A_2A_3$ is an acute-angled triangle. The foot of the altitude from A_i is K_i, and the incircle touches the side opposite A_i at L_i. The line K_1K_2 is reflected in the line L_1L_2. Similarly, the line K_2K_3 is reflected in L_2L_3 and K_3K_1 is reflected in L_3L_1. Show that the three new lines form a triangle with vertices on the incircle. *(Russia)*