

Yugoslav IMO Team Selection Test 1998

Bečej, April 12, 1998.

*Time allowed 180 minutes.
Each problem is worth 25 points.*

1. From a deck of playing cards, four *threes*, four *fours* and four *fives* are selected and put down on a table with the main side up. Players *A* and *B* alternately take the cards one by one and put them on the pile. Player *A* begins. A player after whose move the sum of values of the cards on the pile is

- (a) greater than 34;
- (b) greater than 37;

loses the game. Which player has a winning strategy?

2. In a convex quadrilateral $ABCD$, the diagonal AC intersects the diagonal BD at its midpoint S . The radii of incircles of triangles ABS, BCS, CDS, DAS are r_1, r_2, r_3, r_4 , respectively. Prove that

$$|r_1 - r_2 + r_3 - r_4| \leq \frac{1}{8}|AB - BC + CD - DA|.$$

3. Prove that there are no positive integers n and $k \leq n$ such that the numbers

$$\binom{n}{k}, \binom{n}{k+1}, \binom{n}{k+2}, \binom{n}{k+3}$$

in this order form an arithmetic progression.