

15-th Balkan Mathematical Olympiad

Nicosia, Cyprus – May 5, 1998

1. Consider the finite sequence $\left[\frac{k^2}{1998} \right]$, $k = 1, 2, \dots, 1997$. How many distinct terms are there in this sequence? *(Greece)*

2. Let $n \geq 2$ be an integer, and let $0 < a_1 < a_2 < \dots < a_{2n+1}$ be real numbers. Prove the inequality

$$\sqrt[n]{a_1} - \sqrt[n]{a_2} + \sqrt[n]{a_3} - \dots + \sqrt[n]{a_{2n+1}} < \sqrt[n]{a_1 - a_2 + a_3 - \dots + a_{2n+1}}.$$

(Romania)

3. Let \mathcal{S} denote the set of points inside or on the border of a triangle ABC , without a fixed point T inside the triangle. Show that \mathcal{S} can be partitioned into disjoint closed segments. *(Yugoslavia)*

4. Prove that the equation $y^2 = x^5 - 4$ has no integer solutions. *(Bulgaria)*