23-rd Iberoamerican Mathematical Olympiad

September 23–24, 2008

First Day

1. The integers $1, 2, \dots, 2008^2$ are written one each square of a 2008×2008 board. For every row and column the difference between the maximal and minimal of the numbers is computed. Let S be the sum of these 4016 numbers. Find the largest possible value for S.

2. Given a triangle ABC, let r be the external bisector of $\angle ABC$. Let P and Q be the feet of perpendiculars from A and C to r. If $CP\cap BA = M$ and $AQ\cap BC = N$, show that MN, r, and AC pass through the same point.

3. Let $P(x) = x^3+mx+n$ be an integer polynomial that for each $x,y \in \mathbb{Z}$ satisfies:

   If $P(x)-P(y)$ is divisible by 107, then $x-y$ is divisible by 107 as well.

   Prove that $107 \mid m$.

Second Day

4. Prove that there are no integers x and y such that

$$x^{2008}+2008! = 21^y.$$

5. Let X, Y, and Z be the points of the sides BC, CA, and AB of $\triangle ABC$. Let A', B', and C' be the circumcenters of $\triangle AZY$, $\triangle BXZ$, and CYX, respectively. Prove that $4S_{A'B'C'} \ge S_{ABC}$ with equality if and only if AA', BB', and CC' pass through the same point.

6. *Biribol* is a game played between two teams of 4 people each (teams are not fixed). Find all the possible values of n for which it is possible to arrange a tournament with n players in such a way that every couple of participants plays a match in opposite teams exactly once.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com