Yugoslav IMO Team Selection Test 1972

Belgrade, 1972

1. Given non-zero real numbers u, v, w, x, y, z, how many different possibilities are there for the signs of these numbers if

$$(u+ix)(v+iy)(w+iz)=i?$$

2. If a convex set of points in the line has at least two diameters, say AB and CD, prove that AB and CD have a common point.

3. Assume that the numbers from the table

$$\begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{array}$$

satisfy the inequality:

$$\sum_{j=1}^{n} |a_{j1}x_1 + a_{j2}x_2 + \cdots + a_{jn}x_n| \leq M,$$

for each choice $x_j = \pm 1$. Prove that

$$|a_{11} + a_{22} + \cdots + |a_{nn}| \leq M.$$

4. Determine the largest integer $k(n)$ with the following properties: There exist $k(n)$ different subsets of a given set with n elements such that each two of them have non-empty intersection.

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