Yugoslav IMO Team Selection Test 1996

Bar, April 14, 1996.

Time allowed 180 *minutes.*
Each problem is worth 25 *points.*

1. Let $\mathscr{F} = \{A_1, A_2, \dots, A_n\}$ be a collection of subsets of the set $S = \{1, 2, \dots, n\}$ satisfying the following conditions:

   (a) any two distinct sets from $\mathscr{F}$ have exactly one element in common;

   (b) each element of S is contained in exactly k of the sets in $\mathscr{F}$.

   Can n be equal to 1996?

2. Let be given a set of 1996 equal circles in the plane, no two of them having common interior points. Prove that there exists a circle touching at most three other circles.

3. The sequence $\{x_n\}$ is given by

$$x_n = \frac{1}{4}\left(\left(2+\sqrt{3}\right)^{2n-1} + \left(2-\sqrt{3}\right)^{2n-1}\right), \qquad n \in \mathbb{N}.$$

   Prove that each x_n is equal to the sum of squares of two consecutive integers.

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