

# Yugoslav IMO Team Selection Test 1995

Vrbas, April 16, 1995.

*Time allowed 180 minutes.  
Each problem is worth 25 points.*

1. Determine all triples  $(x, y, z)$  of positive rational numbers with  $x \leq y \leq z$  such that  $x + y + z$ ,  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$  and  $xyz$  are natural numbers.
2. A natural number  $n$  has exactly 1995 units in its binary representation. Show that  $n!$  is divisible by  $2^{n-1995}$ .
3. Let  $SABCD$  be a pyramid with the vertex  $S$  whose all edges are equal. Points  $M$  and  $N$  on the edges  $SA$  and  $BC$  respectively are such that  $MN$  is perpendicular to both  $SA$  and  $BC$ . Find the ratios  $SM : MA$  and  $BN : NC$ .