

# Yugoslav IMO Team Selection Test 1987

Titov Vrbas, April 1987.

1. Let  $x_0 = a$ ,  $x_1 = b$  and  $x_{n+1} = 2x_n - 9x_{n-1}$  for each  $n \in \mathbb{N}$ , where  $a, b$  are integers. Find the necessary and sufficient condition on  $a$  and  $b$  for the existence of an  $x_n$  which is a multiple of 7.
2. Let  $f(x) = \frac{\sqrt{2+\sqrt{2}}x + \sqrt{2-\sqrt{2}}}{-\sqrt{2-\sqrt{2}}x + \sqrt{2+\sqrt{2}}}$ . Find  $\underbrace{f(f(\dots f(x)\dots))}_{1987 \text{ times}}$ .
3. Let be given lines  $a, b, c$  in the space, no two of which are parallel. Suppose that there exist planes  $\alpha, \beta, \gamma$  which contain  $a, b, c$  respectively, which are perpendicular to each other. Construct the intersection point of these three planes. (A space construction permits drawing lines, planes and spheres and translating objects for any vector.)