Yugoslav IMO Team Selection Test 1985

Cetinje, April 1985

1. Suppose each element $i \in S = \{1, 2, \dots, n\}$ is assigned a nonempty set $S_i \in S$ so that the following conditions are fulfilled:

   (i) for any $i, j \in S$, if $j \in S_i$ then $i \in S_j$;

   (ii) for any $i, j \in S$, if $|S_i| = |S_j|$ then $S_i \cap S_j = \emptyset$.

   Prove that there exists $k \in S$ for which $|S_k| = 1$.

2. Let $ABCD$ be a parallelogram and let E be a point in the plane such that $AE \perp AB$ and $BC \perp EC$. Show that either $\angle AED = \angle BEC$ or $\angle AED + \angle BEC = 180^\circ$.

3. If a, b, c, d are positive real numbers, prove that

$$\frac{a}{b+c} + \frac{b}{c+d} + \frac{c}{d+a} + \frac{d}{a+b} \geq 2.$$

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