Yugoslav IMO Team Selection Test 1973

Belgrade, 1973

1. All sides of a rectangle are odd positive integers. Prove that there does not exist a point inside the rectangle whose distance to each of the vertices is an integer.

2. A circle k is drown using a given disc (e.g. a coin). A point A is chosen on k. Using just the given disc, determine the point B on k so that AB is a diameter of k. (You are allowed to choose an arbitrary point in one of the drawn circles, and using the given disc it is possible to construct either of the two circles that passes through the points at a distance that is smaller than the radius of the circle.)

3. Several points are denoted on a white piece of paper. The distance between each two of the points is greater than 2. A drop of an ink was sprinkled over the paper covering an area smaller than π. Prove that there exists a vector $\overrightarrow{v}$ with $\overrightarrow{v} < 1$, such that after translating all of the points by v none of them is covered in ink.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com