Serbia and Montenegro Team Selection Test 2004

Šabac, April 18, 2004

Time allowed 3 *hours.*
Each problem is worth 25 *points.*

1. Let $ABCD$ be a square and γ be a circle with diameter AB. For an arbitrary point P on side CD, segments AP and BP meet γ again at points M and N, respectively, and lines DM and CN meet at point Q. Prove that $Q \in \gamma$ and that $AQ : QB = DP : PC$. *(D. Đukić)*

2. Let a, b, c be real numbers such that $abc = 1$. Prove that at most two of the numbers
$$2a - \frac{1}{b}, \quad 2b - \frac{1}{c}, \quad 2c - \frac{1}{a}$$
are greater than 1. *(Đ. Dugošija)*

3. Let $P(x)$ be the polynomial of degree n whose roots are $i-1, i-2, \ldots, i-n$ (where $i^2 = -1$), and let $R(x)$ and $S(x)$ be the polynomials with real coefficients such that
$$P(x) = R(x) + iS(x).$$
Show that the polynomial R has n real roots. *(R. Stanojević)*

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