Vietnamese IMO Team Selection Test 1996

First Day – May 17

1. Let S be a set of $3n$ points in the plane ($n > 1$), no thre of which are collinear, such that the distance between any two is at most 1. Prove that one construct n pairwise disjoint triangles whose all vertices are in S and whose sum of the areas is less than 1/2.

2. For a positive integer n, let $f(n)$ be the greatest integer for which $2^{f(n)}$ divides the number
$$\sum_{i=0}^{[\frac{n-1}{2}]} \binom{n}{2i+1} 3^i.$$
Find all positive integers n such that $f(n) = 1996$.

3. If a, b, c are real numbers with the sum 1, find the minimum value of
$$f(a,b,c) = (a+b)^4 + (b+c)^4 + (c+a)^4 - \frac{4}{7}(a^4 + b^4 + c^4).$$

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4. For any point M in the plane of a triangle ABC, let $f(M), g(M), h(M)$ be the reflections of M in BC, CA, AB, respectively. Determine all points M for which the segment with endpoints at M and $f(g(h(M)))$ has the minimum length $d(f,g,h)$. Also prove that $d(f,g,h) = d(f,h,g) = \cdots = d(h,g,f)$.

5. Some persons are invited to a party. None of the persons is acquainted to exactly 56 others and any two non-acquainted persons have a common acquaintance among the other persons. Can the number of invited persons be equal to 65?

6. A sequence (x_n) is defined by $x_0 = \sqrt{1996}$ and $x_{n+1} = \dfrac{a}{1 + x_n^2}$ for $n \geq 0$, where a is a real number. Find all values of a for which (x_n) has a finite limit as n tends to infinity.

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