Vietnamese IMO Team Selection Test 1996

First Day – May 17

1. Let \( S \) be a set of \( 3n \) points in the plane \( (n > 1) \), no three of which are collinear, such that the distance between any two is at most 1. Prove that one construct \( n \) pairwise disjoint triangles whose all vertices are in \( S \) and whose sum of the areas is less than \( 1/2 \).

2. For a positive integer \( n \), let \( f(n) \) be the greatest integer for which \( 2^{f(n)} \) divides the number

\[
\sum_{i=0}^{\lfloor n/2 \rfloor} \binom{n}{2i+1} 3^i.
\]

Find all positive integers \( n \) such that \( f(n) = 1996 \).

3. If \( a, b, c \) are real numbers with the sum 1, find the minimum value of

\[
f(a, b, c) = (a + b)^4 + (b + c)^4 + (c + a)^4 - \frac{4}{7}(a^4 + b^4 + c^4).
\]

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4. For any point \( M \) in the plane of a triangle \( ABC \), let \( f(M), g(M), h(M) \) be the reflections of \( M \) in \( BC, CA, AB \), respectively. Determine all points \( M \) for which the segment with endpoints at \( M \) and \( f(g(h(M))) \) has the minimum length \( d(f, g, h) \). Also prove that \( d(f, g, h) = d(f, h, g) = \cdots = d(h, g, f) \).

5. Some persons are invited to a party. None of the persons is acquainted to exactly 56 others and any two non-acquainted persons have a common acquaintance among the other persons. Can the number of invited persons be equal to 65?

6. A sequence \( (x_n) \) is defined by \( x_0 = \sqrt{1996} \) and \( x_{n+1} = \frac{a}{1 + x_n} \) for \( n \geq 0 \), where \( a \) is a real number. Find all values of \( a \) for which \( (x_n) \) has a finite limit as \( n \) tends to infinity.