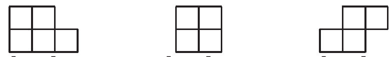
Vietnamese IMO Team Selection Test 1993

First Day – May 4

1. Three kinds of tiles are given:



kind 1 *kind 2* *kind 3*

A rectangle 1993×2000 is tiled with m tiles of kind 1, n tiles of kind 2 and p kinds of kind 3. Find the maximum possible value of $n+p$.

2. A sequence (a_n) is defined by $a_1 = 1$ and $a_{n+1} = a_n + a_n^{-1/2}$. Find all real numbers α such that the sequence a_n^{α}/n converges to a nonzero limit.

3. If x_1, x_2, x_3, x_4 are real numbers with $\frac{1}{2} \leq x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq 1$, find the maximum and minimum values of

$$A = (-2x_1 + x_2)^2 + (x_1 - 2x_2 + x_3)^2 + (x_2 - 2x_3 + x_4)^2 + (x_3 - 2x_4)^2.$$

Second Day – May 5

4. Let H, O, I be respectively the orthocenter, circumcenter and incenter of a triangle ABC. Prove that $2IO \geq OH$. When does equality hold?

5. Let $\varphi(n)$ denote the Euler function. Find all integers $k > 1$ with the following property: for any positive integer a, the sequence x_n defined by $x_0 = a$ and $x_{n+1} = k\varphi(x_n)$ for $n \geq 0$ is bounded.

6. Find the largest n satisfying the following condition: There exists a graph with n vertices, each vertex having degree at most 4, such that any two vertices A and B are either adjacent or there is another vertex C which is adjacent to both A and B.

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