Vietnamese IMO Team Selection Test 1992

First Day

1. Let $n > 1$ and m be two natural numbers. Find the smallest positive integer k with the following property: Among any k integers $a_1, a_2, \dots, a_k$ for which none of the differences $a_i - a_j$ ($i \neq j$) is divisible by n, there exist two numbers a_p, a_s ($p \neq s$) such that $m + a_p - a_s$ is divisible by n.

2. A non-constant polynomial $f(x)$ with real coefficients is given. Show that for every $c > 0$ there is a positive integer n_0 with the following property: For every monic polynomial $P(x)$ of degree n_0 with real coefficients the number of integers x with $|f(P(x))| \leq c$ does not exceed the degree of $P(x)$.

3. A scalene triangle ABC with sides $BC = a$, $CA = b$, $AB = c$ is given in the plane. Points A', B', C' in the plane satisfy the following conditions:

   (i) Pairs of points A and A', B and B', C and C' either all lie on one side or all lie on different sides of lines BC, CA, AB, respectively;

   (ii) Triangles $A'BC$, $B'CA$, and $C'AB$ are similar and isosceles.

   Find all possible values of $\angle A'BC$ in terms of a, b, c, assuming that AA', BB', CC' are not sides of a non-degenerate triangle.

Second Day

4. A finite set of circles is given in the plane such that any two circles are either exterior to each other or externally tangent, and every circle is tangent to at most 6 other circles. Every circle not touching 6 other circles is assigned a real number. Prove that there is at most one way of assigning to each of the remaining circles a real number that is equal to the arithmetic mean of the numbers assigned to the six circles touching it.

5. Find all pairs of positive integers (x, y) such that $x^2 + y^2 - 5xy + 5 = 0$.

6. The participants of a scientific conference speak $2n$ languages in total ($n \geq 2$). Each participant speaks exactly two languages and every two participants can talk to each other in at most one language. Assume that for every integer k with $1 \leq k \leq n-1$ there are at most $k-1$ languages each of which is spoken by at most k participants. Show that one can choose a group of $2n$ participants such that each of the $2n$ languages is spoken by exactly two participants in this group.

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