Vietnamese IMO Team Selection Test 2007

First Day - April 7

1. Let be given two distinct n-element sets A and B of positive numbers with the same sum of elements. Show that there is an $n \times n$ array of nonnegative real numbers such that

   (a) The set of the sums of elements in the rows equals A;

   (b) The set of the sums of elements in the columns equals B;

   (c) There are at least $(n-1)^2 + k$ zeros in the array, where $k = |A \cap B|$.

2. For an acute triangle ABC with incircle (I), let (K_A) be the circle with $AK_A \perp BC$ that passes through A and touches (I) internally at some point A_1. We similarly define points B_1 and C_1.

   (a) Prove that AA_1, BB_1 and CC_1 are concurrent at some point P.

   (b) Let $(J_A), (J_B), (J_C)$ be the circles symmetric to the excircles (I_A), (I_B), (I_C) of triangle ABC with respect to the midpoints of BC, CA, AB, respectively. Prove that point P has the same power with respect to each of the circles (J_A), (J_B), (J_C).

3. If α, β, γ are the angles of a triangle, find the minimum of the expression

$$\frac{\cos^2\frac{\alpha}{2}\cos^2\frac{\beta}{2}}{\cos^2\frac{\gamma}{2}} + \frac{\cos^2\frac{\beta}{2}\cos^2\frac{\gamma}{2}}{\cos^2\frac{\alpha}{2}} + \frac{\cos^2\frac{\gamma}{2}\cos^2\frac{\alpha}{2}}{\cos^2\frac{\beta}{2}}.$$

Second Day - April 8

1. Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that for all real x

$$f(x) = f\left(x^2 + \frac{x}{3} + \frac{1}{9}\right).$$

2. Let A be a 2007-element subset of $\{1, 2, \dots, 4014\}$ such that no two distinct elements of A divide each other. Find the smallest possible value of the minimum element m_A of set A.

3. The vertices $A_1, A_2, \dots, A_9$ of a regular 9-gon have been partitioned into three-element subsets S_1, S_2, S_3. Prove that there always exist different points $A, B \in S_1$, $C, D \in S_2$, $E, F \in S_3$ such that $AB = CD = EF$.

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The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com
Typed in LaTeX by Ercole Suppa