Vietnamese IMO Team Selection Test 2001

First Day – Hanoi, May 8

1. The sequence of integers (a_n) is defined by $a_0 = 1$ and
$$a_n = a_{n-1} + a_{[n/3]}, \qquad \text{for every } n \in \mathbb{N}.$$
Prove that for each prime number $p \leq 13$ there exists k such that a_k is divisible by p.

2. Two circles intersect each other at points A and B. Let l be a common tangent of the two circles, touching them at P and T. The tangents to the circumcircle of triangle APT at P and T meet at S. Let H be the reflection of point B across the line l. Prove that A,S,H are collinear.

3. There are 42 members in a club. Among any 31 of them, there is a pair consisting of a man and a woman who know each other. Prove that there are at least 12 disjoint man-woman pairs who know each other.

Second Day – Hanoi, May 9

4. Let x,y,z be positive real numbers such that $21xy + 2yz + 8zx \leq 12$. Find the minimum value of
$$f(x,y,z) = \frac{1}{x} + \frac{2}{y} + \frac{3}{z}.$$

5. Let $n > 1$ be an integer. Denote by $\mathscr{A}$ the set of points (x,y,z), where $x,y,z \in \{1,2,\dots,n\}$. Some points in $\mathscr{A}$ are colored in such a manner that if point $M(x_0,y_0,z_0)$ is colored, then point $N(x_1,y_1,z_1)$ with $x_1 \leq x_0$, $y_1 \leq y_0$, $z_1 \leq z_0$ is not colored. Find, with proof, the maximum possible nummber of colored points.

6. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of positive integers satisfying the condition
$$0 < a_{n+1} - a_n \leq 2001 \qquad \text{for all } n \in \mathbb{N}.$$
Prove that there exist infinitely many pairs of positive integers (p,q) such that $p < q$ and a_p divides a_q.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com