1. Solve the system of equations
\[(1 + 4^{2+y})5^{1-2x+y} = 1 + 2^{2x-y+1},
\]
\[y^3 + 4x + 1 + \ln(y^2 + 2x) = 0.
\]

2. Let \(A', B', C'\) be the midpoints of the arcs \(BC, CA, AB\) of the circumcircle of a triangle \(ABC\), not containing \(A, B, C\), respectively. The sides \(BC, CA\) and \(AB\) meet the pairs of segments \(C'A', A'B', B'C', C'A'\) at \(M, N; P, Q\) and \(R, S\) respectively. Prove that \(MN = PQ = RS\) if and only if the triangle \(ABC\) is equilateral.

3. The sequences \((x_n)\) and \((y_n)\) are defined recursively as follows:
\[x_0 = 1, \quad x_1 = 4, \quad x_{n+2} = 3x_{n+1} - x_n,\]
\[y_0 = 1, \quad y_1 = 2, \quad y_{n+2} = 3y_{n+1} - y_n, \quad \text{for all } n \geq 0.
\]
(a) Prove that \(x_n^2 - 5y_n^2 + 4 = 0\) for all \(n \geq 0\).
(b) Suppose that \(a, b\) are positive integers satisfying \(a^2 - 5b^2 + 4 = 0\). Prove that there exists \(k \geq 0\) such that \(x_k = a\) and \(y_k = b\).

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4. Let \(a, b, c\) be positive real numbers such that \(abc + a + c = b\). Find the greatest possible value of
\[P = \frac{2}{a^2 + 1} - \frac{2}{b^2 + 1} + \frac{3}{c^2 + 1}.
\]

5. Let \(Ox, Oy, Oz, Ot\) be rays in space, not all in the same plane, such that the angles between any two of them have the same measure.

(a) Determine this common measure.
(b) Let a ray \(Or\), different from these four rays, form angles \(\alpha, \beta, \gamma, \delta\) with \(Ox, Oy, Oz, Ot\), respectively. Denote
\[p = \cos \alpha + \cos \beta + \cos \gamma + \cos \delta, \quad q = \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta.
\]
Prove that \(p\) and \(q\) remain constant when \(Or\) varies.

6. Let \(\mathbb{T}\) denote the set of nonnegative integers not greater than 1999. Find all functions \(f: \mathbb{N}_0 \to \mathbb{T}\) which satisfy
\[f(t) = t \quad \text{for all } t \in \mathbb{T},
\]
\[f(m + n) = f(f(m) + f(n)) \quad \text{for all } m, n \in \mathbb{N}.
\]