33-rd Vietnamese Mathematical Olympiad 1995

First Day – March 2

1. Solve the equation $x^3 - 3x^2 - 8x + 40 - 8\sqrt[4]{4x+4} = 0$.

2. The sequence (a_n) is defined by $a_0 = 1$, $a_1 = 3$ and

$$a_{n+2} = \begin{cases} a_{n+1} + 9a_n & \text{for } n \text{ even;} \\ 9a_{n+1} + 5a_n & \text{for } n \text{ odd.} \end{cases}$$

   (a) Prove that $a_{1995}^2 + a_{1996}^2 + \cdots + a_{2000}^2$ is divisible by 20.

   (b) Prove that a_{2n+1} is not a perfect square for any $n \in \mathbb{N}$.

3. Let AD, BE, CF be the altitudes of a non-equilateral triangle ABC, and let A', B', C' be the points on AD, BE, CF respectively such that

$$\overrightarrow{AA'} = k\overrightarrow{AD}, \quad \overrightarrow{BB'} = k\overrightarrow{BE}, \quad \overrightarrow{CC'} = k\overrightarrow{CF},$$

   where k is a real number. Find all values of k such that $\triangle A'B'C'$ is similar to $\triangle ABC$ for any ABC.

Second Day – March 3

4. In a tetrahedron $ABCD$, points A', B', C', D' are the circumcenters of the triangles BCD, CDA, DAB, ABC respectively. Prove that the four planes which pass through A, B, C, D and are perpendicular to $C'D', D'A', A'B', B'C'$ respectively have a common point P. If P is the circumcenter of $ABCD$, is $ABCD$ necessarily regular?

5. Find all polynomials $P(x)$ with real coefficients such that for all $a > 1995$, the number of real roots of $P(x) - a$ (with multiplicities) is greater than 1995 and all these roots are greater than 1995.

6. The vertices of a regular $2n$-gon ($n \geq 2$) are colored by n colors so that:

   (i) each vertex is aolored by exactly one color, and

   (ii) each color is used for two non-adjacent vertices.

   Two colorings are called *equivalent* if one is obtained from the other by a rotation about the center of the polygon. Find the number of pairwise non-equivalent colorings.

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