1. Solve the system of equations in $x, y, z$:

\[
\begin{align*}
    x^3 + 3x - 3 + \ln(x^2 - x + 1) &= y; \\
    y^3 + 3y - 3 + \ln(y^2 - y + 1) &= z; \\
    z^3 + 3z - 3 + \ln(z^2 - z + 1) &= x.
\end{align*}
\]

2. Let $ABC$ be a triangle and let $A', B', C'$ be points symmetric to $A, B, C$ with respect to $BC, CA, AB$, respectively. Find the necessary and sufficient condition for $\triangle ABC$ so that $\triangle A'B'C'$ is equilateral.

3. Given $0 < a < 1$, a sequence $(x_n)$ is defined by $x_0 = a$ and

\[
x_n = \frac{4}{\pi^2} \left( \arccos x_{n-1} + \frac{\pi}{2} \right) \arcsin x_{n-1}, \quad \text{for } n = 1, 2, \ldots
\]

Prove that this sequence converges and find its limit.

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4. Let be given a convex polygon $A_0A_1 \ldots A_n$ ($n > 2$). Initially, there are $n$ stones at $A_0$. In each move we can choose two vertices $A_i$ and $A_j$ (not necessarily different), take a stone from each of them and put a stone onto a vertex adjacent to $A_i$ and onto a vertex adjacent to $A_j$. Find all values of $n$ such that after finitely many such moves we can achieve that each vertex except $A_0$ contains exactly one stone.

5. A sphere with center $O$ and radius $r$ is given. Two planes $\pi$ and $\theta$ passing through $O$, perpendicular to each other, intersect the sphere in circles $T_\pi$ and $T_\theta$, respectively. Find the locus of the orthocenter $H$ of an orthogonal tetrahedron $ABCD$ (i.e. with $AB \perp CD, AC \perp BD$ and $AD \perp BC$) with $A$ on $T_\theta$ and $B, C, D$ on $T_\pi$.

6. Do there exist polynomials $P(x), Q(x), T(x)$ with positive integer coefficients such that

\[
T(x) = (x^2 - 3x + 3)P(x), \\
P(x) = \left( \frac{1}{20} x^2 - \frac{1}{15} x + \frac{1}{12} \right) Q(x).
\]