# 27-th Vietnamese Mathematical Olympiad 1989

*First Day*

1. Let $n$ and $N$ be natural number. Prove that for any $\alpha$, $0 \le \alpha \le N$, and any real $x$, it holds that
$$\left|\sum_{k=0}^{n} \frac{\sin((\alpha+k)x)}{N+k}\right| \le \min\left((n+1)|x|, \frac{1}{N|\sin\frac{x}{2}|}\right).$$

2. The Fibonacci sequence is defined by $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for $n > 1$. Let $f(x) = 1985x^2 + 1956x + 1960$. Prove that there exist infinitely many natural numbers $n$ for which $f(F_n)$ is divisible by 1989. Does there exist $n$ for which $f(F_n) + 2$ is divisible by 1989?

3. A square $ABCD$ of side length 2 is given on a plane. The segment $AB$ is moved continuously towards $CD$ until $A$ and $C$ coincide with $C$ and $D$ respectively. Let $S$ be the area of the region formed by the segment $AB$ while moving. Prove that $AB$ can be moved in such a way that $S < \dfrac{5\pi}{6}$.

*Second Day*

4. Are there integers $x, y$, not both divisible by 5, such that
$$x^2 + 19y^2 = 198 \cdot 10^{1989}?$$

5. The sequence of polynomials $(P_n(x))$ is defined inductively by $P_0(x) = 0$ and $P_{n+1}(x) = P_n(x) + \dfrac{x - P_n(x)^2}{2}$. Prove that for any $x \in [0,1]$ and any natural number $n$ it holds that
$$0 \le \sqrt{x} - P_n(x) \le \frac{2}{n+1}.$$

6. Let be given a parallelepiped $ABCDA'B'C'D'$. Show that if a line $d$ intersects three of the lines $AB', BC', CD', DA'$, then it intersects also the fourth line.