23-rd Vietnamese Mathematical Olympiad 1985

First Day – February 25

1. Find all pairs (x,y) of integers such that $x^3 - y^3 = 2xy + 8$.

2. Find all functions $f : \mathbb{Z} \to \mathbb{R}$ which satisfy:

   (i) $f(x)f(y) = f(x+y) + f(x-y)$ for all integers x, y;

   (ii) $f(0) \neq 0$;

   (iii) $f(1) = 5/2$.

3. A parallelepiped with the side lengths a, b, c is cut by a plane through its intersection of diagonals which is perpendicular to one of these diagonals. Calculate the area of the intersection of the plane and the parallelepiped.

Second Day – February 26

4. Let a, b and m be positive integers. Prove that there exists a positive integer n such that $(a^n - 1)b$ is divisible by m if and only if $\gcd(ab, m) = \gcd(b, m)$.

5. Find all real values of parameter a for which the equation in x

$$16x^4 - ax^3 + (2a+17)x^2 - ax + 16 = 0$$

   has four solutions which form an arithmetic progression.

6. A triangular pyramid $OABC$ with base ABC has the property that the lengths of the altitudes from A, B and C are not less than $(OB+OC)/2$, $(OC+OA)/2$ and $(OA+OB)/2$, respectively. Given that the area of ABC is S, calculate the volume of the pyramid.

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