21-st Vietnamese Mathematical Olympiad 1983

First Day

1. Are there positive integers a, b with $b \geq 2$ such that $2^a + 1$ is divisible by $2^b - 1$?

2. (a) Prove that $\sqrt{2}(\sin t + \cos t) \geq 2\sqrt[4]{\sin 2t}$ for $0 \leq t \leq \pi/2$.

   (b) Find all y, $0 < y < \pi$, such that $1 + \dfrac{2\cot 2y}{\cot y} \geq \dfrac{\tan 2y}{\tan y}$.

3. A triangle ABC and a positive number k are given. Find the locus of a point M inside the triangle such that the projections of M on the sides of $\triangle ABC$ form a triangle of area k.

Second Day

4. Show that it is possible to express 1 as a sum of 6, and as a sum of 9 reciprocals of odd positive integers. Generalize the problem.

5. Decide whether S_n or T_n is larger, where

$$S_n = \sum_{k=1}^{n} \frac{k}{(2n-2k+1)(2n-k+1)}, \quad T_n = \sum_{k=1}^{n} \frac{1}{k}.$$

6. Let be given a tetrahedron whose any two opposite edges are equal. A plane varies so that its intersection with the tetrahedron is a quadrilateral. Find the positions of the plane for which the perimeter of this quadrilateral is minimum, and find the locus of the centroid for those quadrilaterals with the minimum perimeter.

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