20-th Vietnamese Mathematical Olympiad 1982

First Day

1. Determine a quadric polynomial with integral coefficients whose roots are \( \cos 72^\circ \) and \( \cos 144^\circ \).

2. For a given parameter \( m \), solve the equation

\[
x(x + 1)(x + 2)(x + 3) + 1 - m = 0.
\]

3. Let be given a triangle \( ABC \). Equilateral triangles \( BCA_1 \) and \( BCA_2 \) are drawn so that \( A \) and \( A_1 \) are on one side of \( BC \), whereas \( A_2 \) is on the other side. Points \( B_1, B_2, C_1, C_2 \) are analogously defined. Prove that

\[
S_{ABC} + S_{A_1B_1C_1} = S_{A_2B_2C_2}.
\]

Second Day

4. Find all positive integers \( x, y, z \) such that \( 2^x + 2^y + 2^z = 2336 \).

5. Let \( p \) be a positive integer and \( q, z \) be real numbers with \( 0 \leq q \leq 1 \) and \( q^{p+1} \leq z \leq 1 \). Prove that

\[
\prod_{k=1}^{p} \left| \frac{z - q^k}{z + q^k} \right| \leq \prod_{k=1}^{p} \left| \frac{1 - q^k}{1 + q^k} \right|.
\]

6. Let \( ABCDA'B'C'D' \) be a cube (where \( ABCD \) and \( A'B'C'D' \) are faces and \( AA', BB', CC', DD' \) are edges). Consider the four lines \( AA' \), \( BC \), \( D'C' \) and the line joining the midpoints of \( BB' \) and \( DD' \). Show that there is no line which cuts all the four lines.