19-th Vietnamese Mathematical Olympiad 1981

First Day

1. Prove that a triangle ABC is right-angled if and only if

$$\sin A + \sin B + \sin C = \cos A + \cos B + \cos C + 1.$$

2. Consider the polynomials

$$\begin{aligned} f(p) = \quad & p^{12} - p^{11} + 3p^{10} + 11p^3 - p^2 + 23p + 30; \\ g(p) = \quad & p^3 + 2p + m. \end{aligned}$$

Find all integral values of m for which f is divisible by g.

3. A plane ρ and two points M, N outside it are given. Determine the point A on ρ for which AM/AN is minimal.

Second Day

4. Solve the system of equations

$$\begin{gathered} x^2 + y^2 + z^2 + t^2 = 50; \\ x^2 - y^2 + z^2 - t^2 = -24; \\ xy = zt; \\ x - y + z - t = 0. \end{gathered}$$

5. Let p, q be real numbers with $0 < p < q$ and let $t_1, t_2, \dots, t_n$ be real numbers in the interval $[p, q]$. Denote by A and B the arithmetic means of $t_1, t_2, \dots, t_n$ and of $t_1^2, t_2^2, \dots, t_n^2$, respectively. Prove that

$$\frac{A^2}{B} \geq \frac{4pq}{(p+q)^2}.$$

6. Two circles k_1 and k_2 with centers O_1 and O_2 respectively touch externally at A. Let M be a point inside k_2 and outside the line O_1O_2. Find a line d through M which intersects k_1 and k_2 again at B and C respectively so that the circumcircle of $\triangle ABC$ is tangent to O_1O_2.

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