18-th Vietnamese Mathematical Olympiad 1980

First Day

1. Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be numbers in the interval $[0, 2\pi]$ such that the number $\sum_{i=1}^{n}(1+\cos\alpha_i)$ is an odd integer. Prove that
$$\sum_{i=1}^{n} \sin\alpha_i \geq 1.$$

2. Let $m_1, m_2, \dots, m_k$ be positive numbers with the sum S. Prove that
$$\sum_{i=1}^{k}\left(m_i + \frac{1}{m_i}\right)^2 \geq k\left(\frac{k}{S} + \frac{S}{k}\right)^2.$$

3. Let P be a point inside a triangle $A_1A_2A_3$. For $i = 1,2,3$, line PA_i intersects the side opposite to A_i at B_i. Let C_i and D_i be the midpoints of A_iB_i and PB_i, respectively. Prove that the areas of the triangles $C_1C_2C_3$ and $D_1D_2D_3$ are equal.

Second Day

4. Prove that for any tetrahedron in space, it is possible to find two perpendicular planes such that ratio between the projections of the tetrahedron on the two planes lies in the interval $[\frac{1}{\sqrt{2}}, \sqrt{2}]$.

5. Can the equation $x^3 - 2x^2 - 2x + m = 0$ have three different rational roots?

6. Let be given an integer $n \geq 2$ and a positive real number p. Find the maximum of
$$\sum_{i=1}^{n-1} x_i x_{i+1},$$
where the x_i are nonneagtive real numbers with the sum p.

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