1. Determine the number of solutions of simultaneous equations

\[ x^2 + y^3 = 29, \]
\[ \log_3 x \cdot \log_2 y = 1. \]

2. Given a triangle with acute angle \( \angle BEC \), let \( E \) be the midpoint of \( AB \). Point \( M \) is chosen on the opposite ray of \( EC \) such that \( \angle BME = \angle ECA \). Denote by \( \theta \) the measure of \( \angle BEC \). Express \( MC/AB \) in terms of \( \theta \).

3. Let \( m = 2007^{2008} \). How many natural numbers \( n \) are there such that \( n < m \) and \( n(2n+1)(5n+2) \) divides \( m \)?

4. The sequence of real number \( (a_n) \) is defined by

\[ a_1 = 0, \quad a_2 = 2, \quad a_{n+2} = 2^{-a_n} + \frac{1}{2}, \quad \text{for all } n = 1, 2, 3, \ldots \]

Prove that the sequence has a limit as \( n \) approaches \( +\infty \). Determine the limit.

5. What is the total number of natural numbers divisible by 9 the number of digits of which does not exceed 2008 and at least two of the digits are 9’s?

6. Let \( x, y, z \) be distinct non-negative real numbers. Prove that

\[ \frac{1}{(x-y)^2} + \frac{1}{(y-z)^2} + \frac{1}{(z-x)^2} \geq \frac{4}{xy+yz+zx}. \]

When does equality hold?

7. Let \( ABC \) be a triangle with altitude \( AD \), line \( d \) is perpendicular to \( AD \), and \( M \) is a variable point on \( d \). Let \( E, F \) be the midpoints of \( MB, MC \). The line through \( E \) perpendicular to \( d \) intersects \( AB \) at \( P \), the line through \( F \) perpendicular to \( d \) meets \( AC \) at \( Q \). Prove that the line through \( M \) perpendicular to \( PQ \) has a fixed point as \( M \) varies on the line \( d \).