1. Find all real solutions of the system:
\[
\begin{align*}
\sqrt{x^2 - 2x + 6 \cdot \log_3 (6 - y)} &= x, \\
\sqrt{y^2 - 2y + 6 \cdot \log_3 (6 - z)} &= y, \\
\sqrt{z^2 - 2z + 6 \cdot \log_3 (6 - x)} &= z.
\end{align*}
\]

2. Let \(ABCD\) be a convex quadrilateral and \(M\) be an arbitrary point on line \(AB\). The circumcircles of triangles \(MAD\) and \(MBC\) meet again at \(N\).
   (a) Prove that \(N\) lies on a fixed circle.
   (b) Show that line \(MN\) passes through a fixed point.

3. We are putting marbles onto the squares of an \(m \times n\) board \((m, n > 3)\) according to the following rule. In each step, a marble is put onto each of some four squares that form a figure congruent to the one shown on the image. Is it possible that after finitely many steps each square contains the same number of marbles, if
   (a) \((m, n) = (2004, 2006)\)?
   (b) \((m, n) = (2005, 2006)\)?

Second Day

4. Consider the function \(f(x) = -x + \sqrt{(x + a)(x + b)}\), where \(a, b\) are given distinct positive numbers. Show that for each \(s\) with \(0 < s < 1\) there is exactly one \(\alpha > 0\) such that
\[
f(\alpha) = \sqrt{\frac{a^s + b^s}{2}}.
\]

5. Find all real polynomials \(P(x)\) that satisfy the equality
\[
P(x^2) + x(3P(x) + P(-x)) = P(x)^2 + 2x^2 \quad \text{for all } x.
\]

6. A set \(T\) is called naughty if for any two (not necessarily distinct) elements \(u, v\) of \(T\), \(u + v \not\in T\). Prove that
   (a) a naughty subset of \(S = \{1, 2, \ldots, 2006\}\) has at most 1003 elements;
   (b) every set \(S\) of 2006 positive numbers contains a naughty subset having 669 elements.