1. Solve the system of equations
\[ \begin{align*}
  x^3 + x(y - z)^2 &= 2, \\
  y^3 + y(z - x)^2 &= 30, \\
  z^3 + z(x - y)^2 &= 16.
\end{align*} \]

2. Solve the system of equations
\[ \begin{align*}
  x^3 + 3xy^2 &= -49, \\
  x^2 - 8xy + y^2 &= 8y - 17x.
\end{align*} \]

3. In a triangle \(ABC\), the bisector of \(\angle ACB\) cuts the side \(AB\) at \(D\). An arbitrary circle \(o\) passing through \(C\) and \(D\) meets the lines \(BC\) and \(AC\) at \(M\) and \(N\) (different from \(C\)) respectively.
   (a) Prove that there is a circle \(s\) touching \(DM\) at \(M\) and \(DN\) at \(N\).
   (b) If circle \(s\) intersects the lines \(BC\) and \(CA\) again at \(P\) and \(Q\) respectively, prove that the lengths of the segments \(MP\) and \(NQ\) are constant as \(o\) varies.

4. Let \(ABC\) be an acute triangle with orthocenter \(H\). Point \(P\) distinct from \(B\) and \(C\) is taken on the shorter arc \(BC\) of its circumcircle, and \(D\) is the point such that \(\overrightarrow{AD} = -\overrightarrow{PC}\). Let \(K\) be the orthocenter of \(\triangle ACD\) and \(E\) and \(F\) be the orthogonal projections of \(K\) on \(BC\) and \(AB\). Prove that the line \(EF\) bisects the segment \(HK\).

5. The sequence \((x_n)_{n=1}^{\infty}\) is defined by \(x_1 = 1\) and
\[ x_{n+1} = \frac{(2 + \cos 2\alpha)x_n - \cos^2 \alpha}{(2 - 2\cos 2\alpha)x_n + 2 - \cos 2\alpha} \quad \text{for} \quad n \in \mathbb{N}, \]
where \(\alpha\) is a given real parameter. Find all values of \(\alpha\) for which the sequence \((y_n)\) given by \(y_n = \sum_{k=1}^{n} \frac{1}{2x_{k+1}}\) has a finite limit when \(n \to \infty\) and find that limit.

6. Find the minimum and maximum value of the expression
\[ P = \frac{x^4 + y^4 + z^4}{(x + y + z)^4} \]
where \(x, y, z\) are positive numbers satisfying \((x + y + z)^3 = 32xyz\).

7. Find all triples \((x, y, z)\) of positive integers such that
\[ (x + y)(1 + xy) = 2^z. \]

8. Find the least positive integer \(k\) with the following property: In each \(k\)-element subset of \(A = \{1, 2, \ldots, 16\}\) there exist two distinct elements \(a\) and \(b\) such that \(a^2 + b^2\) is a prime number.
9. Prove that for all integers $n, k$ with $n \geq 2$ and $2n - 3 \leq k \leq \frac{n(n-1)}{2}$ there exist $n$ distinct real numbers $a_1, \ldots, a_n$ such that among their pairwise sums $a_i + a_j$, $1 \leq i < j \leq n$ there are exactly $k$ different numbers.

10. Let $S(n)$ be the sum of decimal digits of a natural number $n$. Find the least value of $S(m)$ if $m$ is an integral multiple of 2003.