2-nd Taiwanese Mathematical Olympiad 1993

Time: 4.5 hours each day.

First Day – April 17, 1993

1. A sequence a_n of positive integers is given by $a_n = [n + \sqrt{n} + 1/2]$. Determine all positive integers which belong to the sequence.

2. Let E and F be distinct points on the diagonal AC of a parallelogram $ABCD$. Prove that, if there exists a circle through E, F tangent to rays BA and BC, then there also exists a circle through E, F tangent to rays DA and DC.

3. Determine all $x, y, z \in \mathbb{N}_0$ such that $7^x + 1 = 3^y + 5^z$.

Second Day – April 20, 1993

4. In the Cartesian plane, let C be a unit circle with center at origin O. For any point Q in the plane distinct from O, define $\widehat{Q}$ to be the intersection of the ray OQ and circle C. Prove that for any $P \in C$ and any $k \in \mathbb{N}$ there exists a lattice point $Q(x, y)$ with $|x| = k$ or $|y| = k$ such that $P\widehat{Q} < \dfrac{1}{2k}$.

5. Assume $A = \{a_1, \dots, a_{12}\}$ is a set of positive integers such that for each positive integer $n \leq 2500$ there is a subset S of A whose sum of elements is n. If $a_1 < a_2 < \cdots < a_{12}$, what is the smallest possible value of a_1?

6. Let m be equal to 1 or 2 and $n < 10799$ be a positive integer. Determine all such n for which
$$\sum_{k=1}^{n} \frac{1}{\sin k \sin(k+1)} = m \frac{\sin n}{\sin^2 1}.$$

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