

Turkish IMO Team Selection Test 2007

First Day

1. Find the number of the connected graphs with 6 vertices. (Vertices are considered to be different)
2. Two different two points A and B and a circle ω that passes through A and B are given. P is a variable point on ω (different from A and B). M is a point such that MP is the bisector of $\angle APB$ (M lies outside of ω) and $MP = AP + BP$. Find the locus of M .
3. Let a, b, c be positive real numbers such that $a + b + c = 1$. Prove that

$$\frac{1}{ab + 2c^2 + 2c} + \frac{1}{bc + 2a^2 + 2a} + \frac{1}{ac + 2b^2 + 2b} \geq \frac{1}{ab + bc + ca}.$$

Second Day

4. Let ABC be an acute-angled triangle and let A_1, B_1, C_1 be the points on BC, CA, AB , respectively such that $\triangle ABC \sim \triangle A_1B_1C_1$. Prove that the orthocenter of $\triangle A_1B_1C_1$ coincides with the circumcenter of $\triangle ABC$.
5. A number n satisfies the following conditions:
 - (i) n is a positive integer;
 - (ii) There are odd integers whose some of squares is equal to n^4 .

Find all such numbers.

6. Find the number of ways to put the numbers 1 and -1 in the cells of 2007×2007 board such that for every square of the board the absolute value of the sum of its numbers is less than or equal to 1.