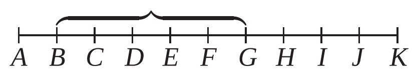
Swedish Mathematical Competition 1993

Final Round

November 20, 1993

1. An integer x has the property that the sums of the digits of x and of $3x$ are the same. Prove that x is divisible by 9.

2. A railway line is divided into ten sections by the stations $A, B, C, D, E, F, G, H, I, J, K$. The length of each section is an integer number of kilometers and the distnace between A and K is 56km. A trip along two successive sections never exceeds 12km, but a trip along three successive sections is at least 17km. What is the distance between B and G?



3. Assume that a and b are integers. Prove that the equation $a^2+b^2+x^2=y^2$ has an integer solution x, y if and only if the product ab is even.

4. To each pair of nonzero real numbers a and b a real number $a*b$ is assigned so that $$a*(b*c)=(a*b)c \quad \text{and} \quad a*a=1 \quad \text{for all } a,b,c.$$
   Solve the equation $x*36=216$.

5. A triangle with sides a, b, c and perimeter $2p$ is given. Is possible, a new triangle with sides $p-a, p-b, p-c$ is formed. The process is then repeated with the new triangle. For which original triangles can this process be repeated indefinitely?

6. For real numbers a and b define $f(x)=\dfrac{1}{ax+b}$. For which a and b are there three distinct real numbers x_1, x_2, x_3 such that $f(x_1)=x_2$, $f(x_2)=x_3$ and $f(x_3)=x_1$?

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