Swedish Mathematical Competition 1989

Final Round

November 18, 1989

1. Let n be a positive integer. Prove that the numbers $n^2(n^2+2)^2$ and $n^4(n^2+2)^2$ are written in base n^2+1 with the same digits but in opposite order.

2. Find all continuous functions f such that $f(x)+f(x^2)=0$ for all real numbers x.

3. Find all positive integers n such that $n^3-18n^2+115n-391$ is the cube of a positive integer.

4. Let $ABCD$ be a regular tetrahedron. Find the positions of point P on the edge BD such that the edge CD is tangent to the sphere with diameter AP.

5. Assume $x_1,x_2,\dots,x_5$ are positive numbers such that $x_1<x_2$ and x_3,x_4, x_5 are all greater than x_2. Prove that if $\alpha>0$, then

$$\frac{1}{(x_1+x_3)^\alpha}+\frac{1}{(x_2+x_4)^\alpha}+\frac{1}{(x_2+x_5)^\alpha}<\frac{1}{(x_1+x_2)^\alpha}+\frac{1}{(x_2+x_3)^\alpha}+\frac{1}{(x_4+x_5)^\alpha}.$$

6. On a circle $4n$ points are chosen ($n\geq 1$). The points are alternately colored yellow and blue. The yellow points are divided into n pairs and the points in each pair are connected with a yellow line segment. In the same manner the blue points are divided into n pairs and the points in each pair are connected with a blue segment. Assume that no three of the segments pass through a single point. Show that there are at least n intersection points of blue and yellow segments.

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