

27-th Swedish Mathematical Competition 1987

Final Round

November 21, 1987

1. Sixteen real numbers are arranged in a magic square of side 4 so that the sum of numbers in each row, column or main diagonal equals k . Prove that the sum of the numbers in the four corners of the square is also k .
2. A circle of radius R is divided into two parts of equal area by an arc of another circle. Prove that the length of this arc is greater than $2R$.
3. Ten closed intervals, each of length 1, are placed in the interval $[0, 4]$. Show that there is a point in the larger interval that belongs to at least four of the smaller intervals.
4. A differentiable function f with $f(0) = f(1) = 0$ is defined on the interval $[0, 1]$. Prove that there exists a point $y \in [0, 1]$ such that

$$|f'(y)| = 4 \int_0^1 |f(x)| dx.$$

5. Show that there exists a positive number t such that for all positive numbers a, b, c, d with $abcd = 1$,

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} + \frac{1}{1+d} > t,$$

and find the largest t with this property.

6. A baker with access to a number of different spices bakes ten cakes. He uses more than half of the different kinds of spices in each cake, but no two of the combinations of spices are exactly the same. Show that there exist three spices a, b, c such that every cake contains at least one of these.