

34-th Spanish Mathematical Olympiad 1998

Second Round

Tarazona

First Part


1. A unit square $ABCD$ with center O is rotated about O by an angle α . Compute the common area of the two squares.
2. Find all four-digit numbers in decimal system that are equal to the cube of the sum of their digits.
3. Let ABC be a triangle. Points D and E are taken on the line BC such that AD and AE are parallel to the respective tangents to the circumcircle at C and B . Prove that

$$\frac{BE}{CD} = \left(\frac{AB}{AC}\right)^2.$$

Second Part

4. Find the tangents of the angles of a triangle knowing that they are positive integers.
5. Find all strictly increasing functions $f : \mathbb{N} \rightarrow \mathbb{N}$ that satisfy

$$f(n + f(n)) = 2f(n) \quad \text{for all } n \in \mathbb{N}.$$

6. Determine the values of n for which an $n \times n$ square can be tiled with pieces of the type .