29-th Spanish Mathematical Olympiad 1993

Second Round
Madrid

First Part

1. There is a reunion of 201 people from 5 different countries. It is known that in each group of 6 people, at least two have the same age. Show that there must be at least 5 people with the same country, age and sex.

2. In the arithmetic triangle below each number (apart from those in the first row) is the sum of the two numbers immediately above.

$$\begin{array}{ccccccccccccccccc} 0 & & 1 & & 2 & & 3 & & 4 & \cdots\cdots & 1991 & & 1992 & & 1993 \\ & 1 & & 3 & & 5 & & 7 & & \cdots\cdots & & 3983 & & 3985 & \\ & & 4 & & 8 & & 12 & & \cdots & \cdots\cdots & \cdots & & 7968 & & \\ & & & & & & & \cdots\cdots\cdots\cdots\cdots\cdots & & & & & & & \end{array}$$

   Prove that the bottom number is a multiple of 1993.

3. Prove that in every triangle the diameter of the incircle is not greater than the radius of the circumcircle.

Second Part

4. Prove that for each prime number distinct from 2 and 5 there exist infinitely many multiples of p of the form $1111\dots1$.

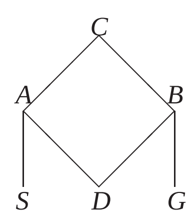
5. Given a 4×4 grid of points, the points at two opposite corners are denoted A and D. We need to choose two other points B and C such that the six pairwise distances of these four points are all distinct.

   (a) How many such quadruples of points are there?

   (b) How many such quadruples of points are non-congruent?

   (c) If each point is assigned a pair of coordinates (x_i, y_i), prove that the sum of the expressions $|x_i - x_j| + |y_i - y_j|$ over all six pairs of points in a quadruple is constant.

6. A game in a casino uses the diagram shown. At the start a ball appears at S. Each time the player presses a button, the ball moves to one of the adjacent letters with equal probability. The game ends when one of the following two things happens:



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(i) The ball returns to S: the player loses.

(ii) The ball reaches G: the player wins.

Find the probability that the player wins and the expected duration of a game.