# 37-th Spanish Mathematical Olympiad 2001

## Second Round

Murcia

*First Part*

1. Prove that the graph of a polynomial $P(x)$ is symmetric with respect to a point $A(a,b)$if and only if there exists a polynomial $Q(x)$ such that
$$P(x) = b + (x-a)Q\left((x-a)^2\right).$$

2. Let $P$ be an interior point of a triangle $ABC$ such that $AP = BP$. On the other two sides of $\triangle ABC$ are externally constructed triangles $BQC$ and $CRA$, similar to triangle $ABP$, with $BQ = QC$ and $CR = RA$. Prove that the points $P, Q, C$ and $R$ are either collinear or vertices of a parallelogram.

3. Five given segments $a_1, a_2, a_3, a_4, a_5$ are such that any three of them are sides of a certain triangle. Show that at least one of these triangles is acute-angled.

*Second Part*

4. The digits 1 to 9 are arranged in a $3\times 3$ board. One computes the sum of the six three-digit numbers: three in the rows left to right and three in the columns top to bottom. Can this sum be equal to 2001?

5. A quadrilateral $ABCD$ is inscribed in a circle of radius 1 whose diameter is $AB$. If the quadrilateral $ABCD$ has an incircle, prove that $CD \le 2(\sqrt{5}-2)$.

6. Find a function $f : \mathbb{N} \to \mathbb{N}$ which satisfies $f(2^s) = f(1)$ for all $s, n \in \mathbb{N}$ and $f(2^s + n) = f(n) + 1$ for $n < 2^s$. Find the maximum value of $f(n)$ for $n \le 2001$ and determine the smallest $n$ for which $f(n) = 2001$.



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