# 33-rd Polish Mathematical Olympiad 1981/82

## Third Round

*First Day*

1. Find a way of arranging $n$ girls and $n$ boys around a round table for which $d_n - c_n$ is maximum, where $d_n$ is the number of girls sitting between two boys and $c_n$ is the number of boys sitting between two girls.

2. In a cyclic quadrilateral $ABCD$ the line passing through the midpoint of $AB$ and the intersection point of the diagonals is perpendicular to $CD$. Prove that either the sides $AB$ and $CD$ are parallel or the diagonals are perpendicular.

3. Find all pairs of positive numbers $(x,y)$ which satisfy the system of equations

$$\begin{aligned} x^2+y^2 &= a^2+b^2 \\ x^3+y^3 &= a^3+b^3 \end{aligned}$$

where $a$ and $b$ are given positive numbers.

*Second Day*

4. On a plane is given a finite set of points. Prove that the points can be covered by open squares $Q_1, Q_2, \dots, Q_n$ such that

$$1 \le \frac{N_j}{S_j} \le 4 \quad \text{for } j = 1, \dots, n,$$

where $N_j$ is the number of points from the set inside square $Q_j$ and $S_j$ is the area of $Q_j$.

5. Integers $x_0, x_1, \dots, x_{n-1}$, $x_n = x_0$, $x_{n+1} = x_1$ satisfy the inequality $(-1)^{x_k} x_{k-1} x_{k+1} > 0$ for $k = 1, 2, \dots, n$. Prove that the difference

$$\sum_{k=0}^{n-1} x_k - \sum_{k=0}^{n-1} |x_k|$$

is divisible by 4.

6. Prove that the sum of dihedral angles in an arbitrary tetrahedron is greater than $2\pi$.



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