

# 24-th Polish Mathematical Olympiad 1972/73

## Third Round

### *First Day*

1. Prove that every polynomial is a difference of two increasing polynomials.
2. Let  $p_n$  denote the probability that, in  $n$  tosses, a fair coin shows the head up 100 consecutive times. Prove that the sequence  $(p_n)$  converges and determine its limit.
3. A polyhedron  $W$  has the following properties:
  - (i) It possesses a center of symmetry.
  - (ii) The section of  $W$  by a plane passing through the center of symmetry and one of its edges is always a parallelogram.
  - (iii) There is a vertex of  $W$  at which exactly three edges meet.

Prove that  $W$  is a parallelepiped.

### *Second Day*

4. A set of segments with the total length less than 1 is given on a line. Prove that every set of  $n$  points on the line can be translated by a vector of length not exceeding  $n/2$ , so that all the obtained points are away from the given segments.
5. Prove that every positive rational number  $m/n$  can be represented as a sum of reciprocals of distinct positive integers.
6. Prove that for every centrally symmetric polygon there is at most one ellipse containing the polygon and having the minimal area.