22-nd Polish Mathematical Olympiad 1970/71

Third Round

First Day

1. Show that if (a_n) is an infinite sequence of distinct positive integers, neither of which contains digit 0 in the decimal expansion, then
$$\sum_{n=1}^{\infty} \frac{1}{a_n} < 29.$$

2. A pool table has the shape of a triangle whose angles are in a rational ratio. A ball positioned at an interior point of the table is hit by a stick. The ball reflects from the sides of the triangle according to the law of reflection. Prove that the ball will move only along a finite number of segments. (It is assumed that the ball does not reach the vertices of the triangle.)

3. A safe is protected with a number of locks. Eleven members of the committee have keys for some of the locks. What is the smallest number of locks necessary so that every six members of the committee can open the safe, but no five members can do it? How should the keys be distributed among the committee members if the number of locks is the smallest?

Second Day

4. Prove that if positive integers x,y,z satisfy the equation $x^n + y^n = z^n$, then $\min(x,y) \geq n$.

5. Find the largest integer A such that, for any permutation of the natural numbers not exceeding 100, the sum of some ten successive numbers is at least A.

6. A regular tetrahedron with unit edge length is given. Prove that:

   (a) There exist four points on the surface S of the tetrahedron, such that the distance from any point of the surface to one of these four points does not exceed $1/2$;

   (b) There do not exist three points with this property.

   The distance between two points on surface S is defined as the length of the shortest polygonal line going over S and connecting the two points.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com