19-th Polish Mathematical Olympiad 1967/68

Third Round

First Day

1. Prove that if a polynomial with integer coefficients takes a value equal to 1 in absolute value at three different integer points, then it has no integer zeros.

2. Show that if at least five persons are sitting at a round table, then it is possible to rearrange them so that everyone has two new neighbors.

3. Given an integer $n > 2$, give an example of a set of n mutually different numbers $a_1, \dots, a_n$ for which the set of their pairwise sums $a_i + a_j$ $(i \neq j)$ contains as few different numbers as possible; also give an example of a set of n different numbers $b_1, \dots, b_n$ for which the set of their pairwise sums $b_i + b_j$ $(i \neq j)$ contains as many different numbers as possible;

Second Day

4. On the plane are chosen $n \geq 3$ points, not all on the same line. Drawing all lines passing through two of these points one obtains k different lines. Prove that $k \geq n$.

5. Given $n \geq 4$ points in the plane such that any four of them are the vertices of a convex quadrilateral, prove that these points are the vertices of a convex polygon.

6. Consider a set of $n > 3$ points in the plane, no three of which are collinear, and a natural number $k < n$. Prove the following statements:

   (a) If $k \leq \frac{n}{2}$, then each point can be connected with at least k other points by segments so that no three segments form a triangle.

   (b) If $k > \frac{n}{2}$ and each point is connected with at least k other points by segments, then some three segments form a triangle.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com