46-th Polish Mathematical Olympiad 1994/95

Second Round

February 17–18, 1995

First Day

1. For a polynomial P with integer coefficients, $P(5)$ is divisible by 2 and $P(2)$ is divisible by 5. Prove that $P(7)$ is divisible by 10.

2. Let $ABCDEF$ be a convex hexagon with $AB = BC$, $CD = DE$ and $EF = FA$. Prove that the lines through C, E, A perpendicular to BD, DF, FB are concurrent.

3. Let a, b, c, d be positive irrational numbers with $a + b = 1$. Show that $c + d = 1$ if and only if $[na] + [nb] = [nc] + [nd]$ for all positive integers n.

Second Day

4. Positive real numbers $x_1, x_2, \dots, x_n$ satisfy the condition $\sum_{i=1}^{n} x_i \le \sum_{i=1}^{n} x_i^2$. Prove the inequality $\sum_{i=1}^{n} x_i^t \le \sum_{i=1}^{n} x_i^{t+1}$ for all real numbers $t > 1$.

5. The incircles of the faces ABC and ABD of a tetrahedron $ABCD$ are tangent to the edge AB in the same point. Prove that the points of tangency of these incircles to the edges AC, BC, AD, BD are concyclic.

6. Determine all positive integers n for which the square $n \times n$ can be cut into squares 2×2 and 3×3 (with the sides parallel to the sides of the big square).

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