13-th Pan-African Mathematical Olympiad

Maputo, Mozambique, 2003

First Day

1. Find all functions $f:\mathbb{N}_0\to\mathbb{N}_0$ satisfying $f(2)=2$, $f(n)<f(n+1)$ and $f(mn)=f(m)f(n)$ for all $m,n\in\mathbb{N}$.

2. The circumference of a circle is arbitrarily divided into four arcs, and the midpoints of these arcs are connected by chords. Prove that two of these chords are perpendicular.

3. Does there exist a base in which all the numbers 10101, 101010101, 1010101010101, ... are prime?

Second Day

4. Does there exist a function $f:\mathbb{N}_0\to\mathbb{N}_0$ such that

$$\underbrace{f(f(\ldots f}_{2003}(n)\ldots))=5n \quad \text{for all } n\in\mathbb{N}_0?$$

5. Find all positive integers n such that 21 divides $2^{2^n}+2^n+1$.

6. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that for all x,y

$$f(x^2)-f(y^2)=(x+y)(f(x)-f(y)).$$

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