11-th Pan-African Mathematical Olympiad

Ouagadougou, Burkina Faso, 2001

First Day

1. Find all positive integers for which $\dfrac{n^3+3}{n^2+7}$ is an integer.

2. Let n be a positive integer. A child builds a wall along a line with n identical cubes. He lays the first cube on the line and at each subsequent step he lays the next cube either on the ground or on top of another cube so that it has a common face with the previous one. How many such distinct walls exist?

3. Let P_0 be a point outside an equilateral triangle ABC such that AP_0C is an isosceles triangle with a right angle at P_0. A grasshopper starts at P_0 and turns around the triangle as follows. From P_0, the grasshopper jumps to the point P_1 symmetric to P_0 with respect to A; then it jumps to the point P_2 symmetric to P_1 with respect to B, then to the point P_3 symmetric to P_2 with respect to C, etc. For each $n \in \mathbb{N}$, compare the distances P_0P_1 and P_0P_n.

Second Day

4. Given a positive integer n and a real number $a > 0$, consider the equation

$$\sum_{i=1}^{n} \left(x_i^2 + (a - x_i)^2 \right) = na^2.$$

   How many solutions $(x_1, \dots, x_n)$ with $0 \le x_i \le a$ for each i does the equation have?

5. Evaluate $\displaystyle\sum_{i=1}^{2001} \left[\sqrt{i}\right]$.

6. Let S_1 be a semicircle with center O and diameter AB. A circle C_1 with center P is tangent to S_1 and to AB at O. A semicircle S_2 with center Q on AB is tangent to S_1 and to C_1. A circle C_2 with center R is internally tangent to S_1 and externally tangent to S_2 and C_1. Prove that $OPRQ$ is a rectangle.

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