The Niels Henrik Abel Contest 1996

Final Round

1. Let S be a circle with center C and radius r, and let $P \neq C$ be an arbitrary point. A line l through P intersects the circle in X and Y. Let Z be the midpoint of XY. Prove that the points Z, as l varies, describe a circle. Find the center and radius of this circle.

2. Prove that $\left[\sqrt{n}+\sqrt{n+1}\right]=\left[\sqrt{4n+1}\right]$ for all $n \in \mathbb{N}$.

3. Per and Kari each have n pieces of paper. They both write down the numbers from 1 to $2n$ in an arbitrary order, one number on each side. Afterwards, they place the pieces of paper on a table showing one side. Prove that they can always place them so that all the numbers from 1 to $2n$ are visible at once.

4. Let $f : \mathbb{N} \to \mathbb{N}$ be a function such that $f(f(1995)) = 95$, $f(xy) = f(x)f(y)$ and $f(x) \leq x$ for all x, y. Find all possible values of $f(1995)$.