Dutch Mathematical Olympiad 1998

Second Round

1. Let σ be a permutation of $\{0,1,2,\ldots,9\}$. Consider the sums of each of the 8 triples of consecutive numbers in this permutation, and let $M(\sigma)$ be the largest of these 8 sums.

   (a) Find a permutation σ_1 such that $M(\sigma_1) = 13$.

   (b) Does there exist a permutation σ_2 such that $M(\sigma_2) = 12$?

2. The base $ABCD$ of a pyramid $TABCD$ is a square whose side has length 4. Find all possible values for the volume of the pyramid, if we know that among the triangles TAB, TBC, TCD, and TDA there exists an isosceles as well as a right-angled triangle.

3. Find positive integers m and n such that $m - n = 189$ and $\text{lcm}\,(m,n) = 133866$.

4. Let $ABCD$ be a convex quadrilateral for which $AC \perp BD$.

   (a) Prove that $AB^2 + CD^2 = BC^2 + DA^2$.

   (b) Let $PQRS$ be a convex quadrilateral such that $PQ = AB$, $QR = BC$, $RS = CD$, and $SP = DA$. Prove that $PR \perp QS$.

5. Find all real solutions of the equation

$$(x+1995)(x+1997)(x+1999)(x+2001)+16=0.$$

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