Dutch Mathematical Olympiad 1992

Second Round

September 18

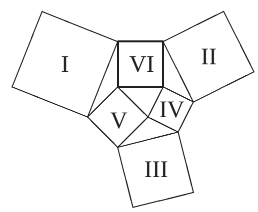
1. Four dice are thrown. What is the probability that the product of the numbers equals 36?

2. In the fraction below and its decimal notation (with period of length 4) every letter represents a digit, and different letters denote different digits. The numerator and denominator are coprime. Determine the value of the fraction:

$$\frac{ADA}{KOK} = 0.SNELSNELSNELSNEL\ldots$$

   Note. Ada Kok is a famous dutch swimmer, and "snel" is Dutch for "fast".

3. Consider the configuration of six squares as shown on the picture. Prove that the sum of the areas of the three outer squares (I, II and III) equals three times the sum of the areas of the three inner squares (IV, V and VI).



4. For every positive integer n, we define $n?$ as $1? = 1$ and

$$n? = \frac{n}{(n-1)?} \quad \text{for } n \geq 2.$$

   Prove that $\sqrt{1992} < 1992? < \frac{4}{3}\sqrt{1992}$.

5. We consider regular n-gons with a fixed circumference 4. Let r_n and a_n respectively be the distnces from the center of such an n-gon to a vertex and to an edge.

   (a) Determine a_4, r_4, a_8, r_8.

   (b) Give an appropriate interpretation for a_2 and r_2.

   (c) Prove that $a_{2n} = \frac{1}{2}(a_n + r_n)$ and $r_{2n} = \sqrt{a_{2n} r_n}$.

   (d) Define $u_0 = 0$, $u_1 = 1$ and

$$u_n = \begin{cases} \frac{1}{2}(u_{n-2} + u_{n-1}) & \text{for } n \text{ even,} \\ \sqrt{u_{n-2}u_{n-1}} & \text{for } n \text{ odd.} \end{cases}$$

   Determine $\lim_{n\to\infty} u_n$.

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