

Dutch Mathematical Olympiad 1983

Second Round

1. A triangle ABC can be divided into two isosceles triangles by a line through A . Given that one of the angles of the triangle is 30° , find all possible values of the other two angles.
2. Prove that if n is an odd positive integer, then the last two digits of $2^{2n}(2^{2n+1} - 1)$ in base 10 are 28.
3. Suppose that a, b, c, p are real numbers with a, b, c not all equal, such that

$$a + \frac{1}{b} = b + \frac{1}{c} = c + \frac{1}{a} = p.$$

Determine all possible values of p and prove that $abc + p = 0$.

4. Within an equilateral triangle of side 15 are 111 points. Prove that it is always possible to cover three of these points by a round coin of diameter $\sqrt{3}$, part of which may lie outside the triangle.