20-th Nordic Mathematical Contest

March 30, 2006

1. Points B and C vary on two fixed rays emanating from a point A such that $AB+AC$ is constant. Show that there is a point $D \neq A$ such that the circumcircle of the triangle ABC passes through D for all possible choices of B and C.

2. Real numbers x, y, z are not all equal and satisfy the condition

$$x+\frac{1}{y}=y+\frac{1}{z}=z+\frac{1}{x}=k.$$

   Find all possible values of k.

3. A sequence (a_n) of positive integers is given by $a_0 = m$ and

$$a_{n+1}=a_n^5+487 \quad \text{for } n \geq 0.$$

   Determine all values of m for which this sequence contains the maximum possible number of squares.

4. Each square of a 100×100 chessboard is painted with one of 100 different colors, so that every color is used exactly 100 times. Show that there exists a row or a column of the chessboard in which at least 10 colors are used.

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